# DESIGN OF CT AND CQ FILTERS USING APPROXIMATION AND OPTIMIZATION

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## ABSTRACT

A new design technique for CT filters has been derived commencing from the well-known Chebyshev all-pole prototype filter. One or more finite frequency poles may be introduced by cross coupling across sets of three nodes, and the filter re-matched by approximate compensation of the element values. Any general optimizer may then be used to obtain a nearly perfect result without undue concern over convergence failures. The method may be combined with a similar previous theory for CQ sections.

**Introduction**. This paper relates to the design of filters having cascaded triplet (CT) or cascaded quadruplet (CQ) sections which are relatively simple to tune compared with filters having "nested" cross-couplings. The latter may have more optimal characteristics but are typically much more difficult to align and tune. Exact design theories using classical synthesis are available [1-3], but recently there has been interest in design techniques based on optimization [4,5]. If optimization is to be used it is preferable to commence from a design, which is as close as possible to the final result, i.e., an approximate theory is desirable. This is available in the case of filters having CQ sections [5] but not for CT sections. It is surprising that the exact design theory of CT sections has preceded the approximate one given in this paper.

Although in many cases it is possible to optimize a design starting from the correct topology but with arbitrarily assigned element values, such methods may not always converge, especially for high-degree filters. Thus the objective function given in [4] does not apply in the case of CQ sections producing real axis transmission zeros used in linear-phase filters. Usually it is preferable to commence from a close approximate design, which is almost certainly guaranteed to converge using any standard optimizer.

In addition the existence of an approximate design may be considered to be of academic interest as well as having practical engineering value. The question of why optimization should be used when exact synthesis is available is also pertinent, but synthesis may not always be simple or readily available, e.g. for diplexers and other complex structures, and many engineers would prefer to use more commonly used procedures.

**Theory of Cascaded CT Sections**. The theory commences from the standard all-pole Chebyshev low-pass prototype filter shown in Fig. 1. This is converted into a band pass filter with band edges $\omega_1$ and $\omega_2$, and normalized mid band frequency

$$\omega_0 = \sqrt{(\omega_1 \omega_2)} \qquad (1)$$

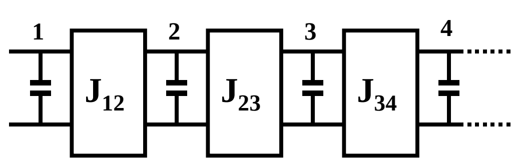


**Fig. 1. Lowpass prototype filter.**

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The admittance inverters may be realized as standard Pi sections of capacitors or inductors [6], but there is a third possibility shown in Fig. 2, which is to use a Pi of parallel LC sections [7,8]. This introduces a pole at

$$\omega_p = 1/\sqrt{(LC)} \qquad (2)$$

and is the basis of the theory. Other ideal admittance inverters are realized as standard inductive or capacitive Pi sections, and the negative shunt circuit elements absorbed into the adjacent main positive shunt elements. The resulting LC circuit may then be converted into one having a CT section using the formulas given in [3]. The procedure guarantees that the pole is produced at the correct frequency, and that the return loss is correct at mid band.

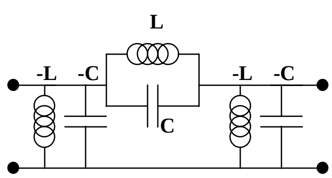


**Fig. 2. Pole-producing admittance inverter.**

**<u>Examples</u>.** The theory will be illustrated by design of 6-section filters having two coincident poles. The lumped element circuit is shown in Fig. 3., and was derived from the prototype of Fig. 1 using three inductive and two pole-producing pi inverters. This topology is appropriate for eventual conversion into a combline filter using Richards' transformation and well-known close approximation techniques. The ripple level is 20 dB return loss, and the return loss bandwidth is 5%. Frequencies are normalized to mid band with band edges at 0.975 and 1.025. The two examples are for poles located at 0.90 and 0.95 respectively, and the comparison between the exact and approximate theories are given in Tables I (a) and I (b). The maximum errors for the element values are approximately 2% for the pole at 0.90 and 7.7% for that at 0.95. Such relatively small error is the reason for the suitability of the method for optimization.

Analysis of the case with the pole at 0.9 showed a return loss of better than 11 dB, and the bandwidth shifted slightly higher in frequency. The analysis for the more severe case with the pole at 0.95, which is much closer to the passband edge, is shown in Fig. 4. Here the return loss has degraded to a worst level of 6.5 dB and the frequency shift has increased.

In both cases the filter optimized within a few seconds using a standard gradient-based optimizer, here "Touchstone". The optimized result for the pole at 0.95 is shown in Fig. 5. The six return loss poles are well resolved, and the 20 dB return loss level is produced quite closely. The result was obtained by retaining the element values of the pole-producing sections, so that the final set of element values differ slightly from those of the exact synthesis.

If a CQ section is desired then the method of [9] is used. Hence filters with mixed CQ and CT sections become feasible, including those where the CQ section produces real axis poles needed for linear phase applications.

**Conclusions.** A new theory for the design of CT filters based on derivation of specific design equations and the element values of the standard Chebyshev all-pole lowpass filter has been described. If the finite attenuation poles are relatively far from the passband then the theory may be quite accurate, but optimization is usually required, and convergence is practically guaranteed since the starting condition is sufficiently close to the final ideal result. The new results complement an earlier result obtained for CQ filters [6], and both CT and CQ sections may be incorporated within a filter design.

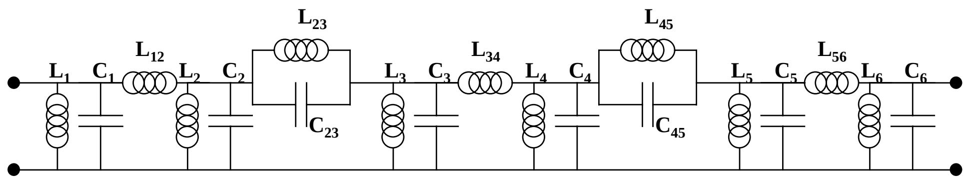


**Fig. 3. N=6 filter with two coincident poles.** *These may be converted into CT sections using matrix operations which removes capacitor 2-3 and introduces one across 1-3, with a similar operation to remove 4-5 and introduce 4-6.*

**Table I. Comparison Between Exact and Approximate Element Values for 6-Section Filters having Pole Pairs at Coincident Frequencies., (a) Poles at 0.90, (b) Poles at 0.95. (See Fig. 3 for Circuit Topology).**

| | EXACT | | APPROXIMATE | |
|---|---|---|---|---|
| | CAPACITANCE | INDUCTANCE | CAPACITANCE | INDUCTANCE |
| $C_1,L_1$ | 19.9182 | 0.0523 | 19.9096 | 0.0525 |
| $L_{12}$ | - | 1.2039 | - | 1.1912 |
| $C_2,L_2$ | 16.753 | 0.0611 | 16.7069 | 0.0607 |
| $C_{23},L_{23}$ | 3.2628 | 0.3784 | 3.2027 | 0.3855 |
| $C_3,L_3$ | 16.7431 | 0.0600 | 16.7069 | 0.0598 |
| $L_{34}$ | - | 1.7595 | - | 1.7213 |
| $C_4,L_4$ | 16.7431 | 0.0600 | 16.7069 | 0.0598 |
| $C_{45},L_{45}$ | 3.2628 | 0.3784 | 3.2027 | 0.3855 |
| $C_5,L_5$ | 16.753 | 0.0611 | 16.7069 | 0.0607 |
| $L_{56}$ | - | 1.2039 | - | 1.1912 |
| $C_6,L_6$ | 19.9182 | 0.0523 | 19.9096 | 0.0525 |

**(a)**

| | EXACT | | APPROXIMATE | |
|---|---|---|---|---|
| | CAPACITANCE | INDUCTANCE | CAPACITANCE | INDUCTANCE |
| $C_1,L_1$ | 19.9201 | 0.0522 | 19.9096 | 0.0525 |
| $L_{12}$ | - | 1.2561 | - | 1.1913 |
| $C_2,L_2$ | 13.4006 | 0.0775 | 13.6684 | 0.0745 |
| $C_{23},L_{23}$ | 6.7210 | 0.1649 | 6.2411 | 0.1775 |
| $C_3,L_3$ | 13.3768 | 0.0757 | 13.6684 | 0.0731 |
| $L_{34}$ | - | 1.9175 | - | 1.7213 |
| $C_4,L_4$ | 13.3768 | 0.0757 | 13.6684 | 0.0731 |
| $C_{45},L_{45}$ | 6.7210 | 0.1649 | 6.2411 | 0.1775 |
| $C_5,L_5$ | 13.4006 | 0.0775 | 13.6684 | 0.0745 |
| $L_{56}$ | - | 1.2561 | - | 1.1913 |
| $C_6,L_6$ | 19.9201 | 0.0522 | 19.9096 | 0.0525 |

**(b)**

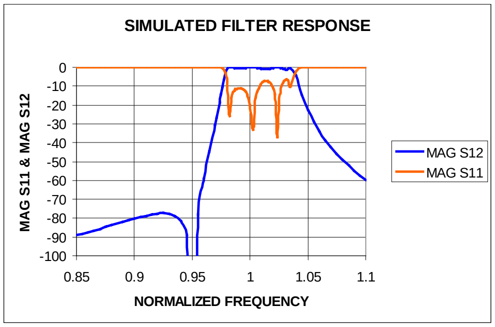


**Fig. 4. Results for the approximate design.**

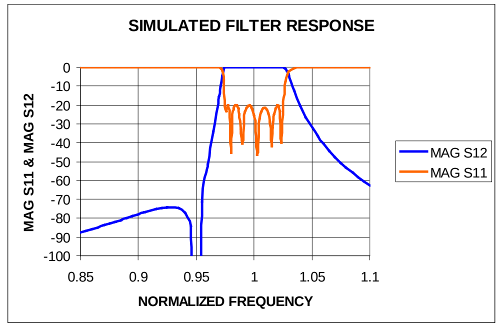


**Fig. 5. Design after optimization.**

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